WP1 Solutions

3. $P(-1,3)$, $Q(1,5)$, $R(-3,7)$

$\overrightarrow{QP} = \langle -1-1, 3-5 \rangle$
$= \langle -2, -2 \rangle$
$\overrightarrow{QP} = -2\hat{\imath} - 2\hat{\jmath}$

7. $P(-1,3)$, $Q(1,5)$, $R(-3,7)$

$2\overrightarrow{PQ} - 2\overrightarrow{PR} = 2\langle 1+1, 5-3 \rangle - 2\langle -3+1, 7-3 \rangle$
$= 2\langle 2,2 \rangle - 2\langle -2,4 \rangle$
$= \langle 4,4 \rangle - \langle -4,8 \rangle$
$= \langle 8,-4 \rangle$
$2\overrightarrow{PQ} - 2\overrightarrow{PR} = 8\hat{\imath} - 4\hat{\jmath}$

13. $P(1,0)$, Q on y-axis, magnitude $= \sqrt{5}$

$\sqrt{5} = \sqrt{(1-0)^2 + (0-y)^2}$
$\sqrt{5} = \sqrt{1+y^2}$
$5 = 1+y^2$
$4 = y^2$
$y = \pm 2$
Since this is above P, need $y > 0$.
So $Q(0,2)$

15. $a = 2\hat{\imath} + \hat{\jmath}$, $b = \hat{\imath} + 3\hat{\jmath}$

a) $a+b = (2\hat{\imath}+\hat{\jmath}) + (\hat{\imath}+3\hat{\jmath})$
$= 3\hat{\imath} + 4\hat{\jmath}$ and $\langle 3,4 \rangle$

b) $a-b = (2\hat{\imath}+\hat{\jmath}) - (\hat{\imath}+3\hat{\jmath})$
$= \hat{\imath} - 2\hat{\jmath}$ and $\langle 1,-2 \rangle$

c) $\|a\| + \|b\| = \sqrt{2^2+1^2} + \sqrt{1^2+3^2}$
$= \sqrt{5} + \sqrt{10}$
$\|a+b\| = \sqrt{3^2+4^2}$
$= \sqrt{25}$
$= 5$
So $\|a\| + \|b\| \geq \|a+b\|$

d) $2a = 2(2\hat{\imath}+\hat{\jmath})$
$= 4\hat{\imath} + 2\hat{\jmath}$
and $\langle 4,2 \rangle$

$-b = -(\hat{\imath}+3\hat{\jmath})$
$= -\hat{\imath} - 3\hat{\jmath}$
and $\langle -1,-3 \rangle$

$2a-b = (4\hat{\imath}+2\hat{\jmath}) + (-\hat{\imath}-3\hat{\jmath})$
$= 3\hat{\imath} - \hat{\jmath}$ and $\langle 3,-1 \rangle$

25. $\|v\| = 7$, $u = \langle 3,4 \rangle$

$\|u\| = \sqrt{3^2+4^2}$
$= \sqrt{25}$
$= 5$
$v = \frac{7}{5}\langle 3,4 \rangle$
$= \langle \frac{21}{5}, \frac{28}{5} \rangle$

45. $v_0 = 70\text{mph}, \theta_0 = 30^\circ$

$v = \langle v_0\cos\theta, v_0\sin\theta\rangle$
$= \langle 70\cos 30^\circ, 70\sin 30^\circ\rangle$
$\langle 35\sqrt{3}, 35\rangle$
$v \approx \langle 60.62, 35\rangle$

77. $P(3,0,2)$ and $Q(-1,-1,4)$

a) $\overrightarrow{PQ} = \langle -1-3, -1-0, 4-2\rangle$
$= \langle -4, -1, 2\rangle$

b) $\overrightarrow{PQ} = -4\hat{i} - \hat{j} + 2\hat{k}$

81. $\overrightarrow{PQ} = \langle 7,-1,3\rangle, P(-2,3,5)$

$\langle 7,-1,3\rangle = \langle x+2, y-3, z-5\rangle$

$x+2=7 \quad y-3=-1 \quad z-5=3$
$x=5 \quad y=2 \quad z=8$

so $Q(5,2,8)$

89. $u = \langle 2\cos t, -2\sin t, 3\rangle, v = \langle 0,0,3\rangle$

$\|u-v\| = \sqrt{(2\cos t-0)^2+(-2\sin t-0)^2+(3-3)^2}$
$= \sqrt{4\cos^2 t + 4\sin^2 t}$
$= 2.$

$\|-2u\| = \sqrt{(-4\cos t)^2 + (4\sin t)^2 + (-6)^2}$
$= \sqrt{16\cos^2 t + 16\sin^2 t + 36}$
$= \sqrt{52}$
$= 2\sqrt{13}$

99. $v = \langle 7,-1,3\rangle, \|u\| = 10$

$\|v\| = \sqrt{7^2 + (-1)^2 + 3^2}$
$= \sqrt{49+1+9}$
$= \sqrt{59}$

$u = \frac{10}{\sqrt{59}}\langle 7,-1,3\rangle$
$= \left\langle \frac{70}{\sqrt{59}}, \frac{-10}{\sqrt{59}}, \frac{30}{\sqrt{59}}\right\rangle$
$u = \left\langle \frac{70\sqrt{59}}{59}, \frac{-10\sqrt{59}}{59}, \frac{30\sqrt{59}}{59}\right\rangle$

115. $F_1 = \langle 10,6,3\rangle, F_2 = \langle 0,4,9\rangle, F_3 = \langle 10,-3,-9\rangle$

$F_1 + F_2 + F_3 = \langle 10+0+10, 6+4-3, 3+9-9\rangle$
$= \langle 20, 7, 3\rangle$

so $F_4 = \langle -20, -7, -3\rangle$

125. $u = \langle 2,2,-1\rangle$, $v = \langle -1,2,2\rangle$

$$u\cdot v = 2(-1)+2(2)+(-1)2$$
$$= -2+4-2$$
$$= 0$$

131. $a = \langle 3,-1\rangle$, $b = \langle -4,0\rangle$

A) $a\cdot b = 3(-4)+(-1)(0) = -12$

$$\|a\| = \sqrt{3^2+(-1)^2} = \sqrt{10}$$
$$\|b\| = \sqrt{(-4)^2+0^2} = 4$$
$$\cos\theta = \frac{-12}{\sqrt{10}\cdot 4}$$
$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{10}}\right)$$
$$\theta = 2.82\text{ rad}$$

B) θ is not acute

145. a orthogonal to $b = \langle 3,4\rangle$

let $a = \langle x,y\rangle$ then
$a\cdot b = 3x+4y$
and $a\cdot b = 0$
So $4y = -3x$
$y = -\frac{3}{4}x$
$a = \langle x, -\frac{3}{4}x\rangle$ for $x\in\mathbb{R}$

171. $u = 4\hat{i}-3\hat{j}$, $v = 3\hat{i}+2\hat{j}$

A) $w = \text{proj}_u v$
$$= \frac{4(3)+(-3)(2)}{4^2+(-3)^2}\langle 4,-3\rangle$$
$$= \frac{12-6}{16+9}\langle 4,-3\rangle$$
$$= \langle 24/25, -18/25\rangle$$

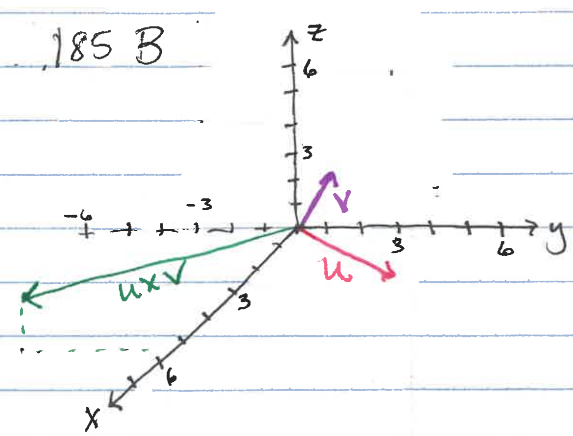
B) $q = \langle 3,2\rangle - \langle 24/25, -18/25\rangle$
$$= \langle 51/25, 68/25\rangle$$

So $v = \left\langle \frac{24}{25}, \frac{-18}{25}\right\rangle + \left\langle \frac{51}{25}, \frac{68}{25}\right\rangle$

177. $\|F\| = 25$, $\|\overrightarrow{PQ}\| = 50$ and $\theta = 20°$

$$W = 25(50)\cos 20°$$
$$= 1175 \text{ ft}\cdot\text{lb}$$

185 B



185. $u = 2\hat{i}+3\hat{j}$; $v = \hat{j}+2\hat{k}$

$$u\times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 6\hat{i}-4\hat{j}+2\hat{k}$$
$$\langle 6,-4,2\rangle$$

189. $u = \langle 3, -1, 2 \rangle$, $v = \langle -2, 0, 1 \rangle$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ -2 & 0 & 1 \end{vmatrix}$$

$$= (-1-0)\hat{i} - (3+4)\hat{j} + (0-2)\hat{k}$$

$$= \langle -1, -7, -2 \rangle$$

$$\|u \times v\| = \sqrt{(-1)^2 + (-7)^2 + (-2)^2}$$

$$= \sqrt{1+49+4}$$

$$= \sqrt{54}$$

$$= 3\sqrt{6}$$

$$W = \frac{u \times v}{\|u \times v\|} = \left\langle \frac{-\sqrt{6}}{18}, \frac{-7\sqrt{6}}{18}, \frac{-\sqrt{6}}{9} \right\rangle$$

199. $u = \langle -1, 0, e^{t} \rangle$ $V = \langle 1, e^{-t}, 0 \rangle$

$$W = u \times v$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & e^{t} \\ 1 & e^{-t} & 0 \end{vmatrix}$$

$$= (0 - e^{0})\hat{i} - (0 - e^{t})\hat{j} + (-e^{-t} - 0)\hat{k}$$

$$= -\hat{i} + e^{t}\hat{j} - e^{-t}\hat{k}$$

$$= \langle -1, e^{t}, -e^{-t} \rangle$$

209. $u = \langle 3, 2, 0 \rangle$, $V = \langle 0, 2, 1 \rangle$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= (2-0)\hat{i} - (3-0)\hat{j} + (6-0)\hat{k}$$

$$= \langle 2, -3, 6 \rangle$$

235. $\|r\| = 12\text{in}$, $F = 10\text{ lb}$, $\theta = 60^\circ$

$$\|\tau\| = 1(10)\sin 60^\circ$$

$$= 8.66 \text{ ft}\cdot\text{lb}$$

237. $\|\tau\| = 100\text{ N}\cdot\text{m}$ $\|r\| = 20\text{cm}$

$V = \langle 0, 1, -2 \rangle$

$u = \langle 0, 1, 0 \rangle$ so

$$\cos\theta = \frac{0(0) + 1(1) + (0)(-2)}{\sqrt{0^2+1^2+0^2} \cdot \sqrt{0^2+1^2+(-2)^2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$100 = 0.2\|F\| \sin\left(\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$$

$$\|F\| = \frac{100}{0.2\sin\left(\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}$$

$$\|F\| = 559\text{N}.$$