WPID Solutions

5. $m=n=2, R=[8,10]\times[9,11]$

$\iint_R f(x,y)\,dA;$

$\Delta x = \frac{10-8}{2} = 1$, $\Delta y = \frac{11-9}{2} = 1$ $\Big\} dA = 1$

$V = [4.5+5.4+6+5.4]\,1$
$= 21.3$

8. 3ft x 3ft hole; $m=n=3$

$\iint_R f(x,y)\,dA$

$\Delta x = \frac{3-0}{3} = 1$, $\Delta y = \frac{3-0}{3} = 1$ $\Big\} dA = 1$

$V = [7+7.5+6.5+6.7+6.5+6+6.5+5+5.6]\,1$
$= 57.3\ ft^3$

17. $\int_{\ln 2}^{\ln 3}\left(\int_0^1 e^{x+y}\,dy\right)dx$

$= \int_{\ln 2}^{\ln 3}\left(e^{x+y}\Big|_0^1\right)dx$
$= \int_{\ln 2}^{\ln 3}(e^{x+1} - e^{x})\,dx$
$= (e^{x+1} - e^{x}\Big|_{\ln 2}^{\ln 3}$
$= (e^{\ln 3+1} - e^{\ln 3}) - (e^{\ln 2+1} - e^{\ln 2})$
$= 3e - 3 - 2e + 2$
$= e - 1$

26. $\int_1^e \int_1^2 x^2 \ln(x)\,dy\,dx$

$= \int_1^e x^2 y \ln(x)\Big|_1^2 dx$
$= \int_1^e [2x^2\ln(x) - x^2\ln(x)]\,dx$
$= \int_1^e x^2 \ln(x)\,dx$ (by IBP)
$= (\frac{1}{3}x^3\ln(x) - \frac{1}{2}x^2\Big|_1^e$
$= \frac{1}{3}e^3 - \frac{1}{2}e^2 - 0 + \frac{1}{2}$
$= \frac{1}{3}e^3 - \frac{1}{2}e^2 + \frac{1}{2}$

29. $\int_0^1\int_1^2 x e^{x+4y}\,dy\,dx$

$= \int_0^1 (\frac{x}{4}e^{x+4y}\Big|_1^2 dx$
$= \int_0^1 (\frac{x}{4}e^{x+8} - \frac{x}{4}e^{x+4})\,dx$
using integration by parts
$= [e^{x+8}(\frac{x}{4}-\frac{1}{4}) - e^{x+4}(\frac{1}{4}-\frac{x}{4})]\Big|_0^1$
$= e^9(0) - e^5(0) - e^8(-1/4) - e^4(\frac{1}{4})$
$= \frac{1}{4}(e^8 - e^4)$

35. $f(x,y) = -x+2y$ $R=[0,1]\times[0,1]$

$\int_0^1\int_0^1 (-x+2y)\,dy\,dx$
$= \int_0^1 (-xy + y^2\Big|_0^1 dx$
$= \int_0^1 (-x+1)\,dx$
$= (-\frac{1}{2}x^2 + x\Big|_0^1$
$= -\frac{1}{2} + 1 - 0$
$= 1/2$

77. $f(x,y) = xy$ and $D = \{(x,y)\mid -1 \le y \le 1,\ y^2-1 \le x \le \sqrt{1-y^2}\}$

$\int_{-1}^1\int_{y^2-1}^{\sqrt{1-y^2}} xy\,dx\,dy$
$= \int_{-1}^1 \frac{1}{2}x^2y\Big|_{y^2-1}^{\sqrt{1-y^2}} dy$
$= \int_{-1}^1 \frac{1}{2}y(1-y^2) - \frac{1}{2}(y)(y^2-1)^2\,dy$
$= \int_{-1}^1 (-\frac{1}{2}y^5 + \frac{1}{2}y^3)\,dy$
$= (-\frac{1}{12}y^6 + \frac{1}{8}y^4\Big|_{-1}^1$
$= (-\frac{1}{12}+\frac{1}{8}) - (-\frac{1}{12}+\frac{1}{8})$
$= 0$

83. $\int_1^2\int_{-u^2-1}^{-u}(8uv)\,dv\,du$

$= \int_1^2 4uv^2 \Big|_{-u^2-1}^{-u} du$

$= \int_1^2 \left[4u^3 - 4u(-u^2-1)^2\right] du$

$= \int_1^2 (-4u^5 - 4u^3 - 4u)\, du$

$= (-\frac{2}{3}u^6 - u^4 - 2u^2 \Big|_1^2$

$= (-\frac{128}{3} - 16 - 8) - (-\frac{2}{3} - 1 - 2)$

$= -63$

95. $f(x,y) = -x+1$ on $(0,0)$, $(0,2)$ and $(2,2)$

$D = \int_0^2\int_x^2 dy\,dx$

$= \int_0^2 y\Big|_x^2 dx$

$= \int_0^2 (2-x)dx$

$= (2x - \frac{1}{2}x^2\Big|_0^2$

$= (4-2) - 0$

$D = 2$

$f_{avg} = \frac{1}{2}\int_0^2\int_x^2 (-x+1)\,dy\,dx$

$= \frac{1}{2}\int_0^2 y(-x+1)\Big|_x^2 dx$

$= \frac{1}{2}\int_0^2 (-2x+2+x^2-x)dx$

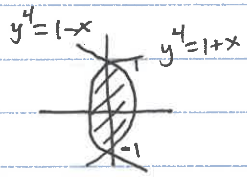
$= \frac{1}{2}\int_0^2 (x^2-3x+2)dx$

$= \frac{1}{2}(\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\Big|_0^2$

$= \frac{1}{2}(\frac{8}{3} - 6 + 4) - \frac{1}{2}(0)$

$f_{avg} = \frac{1}{3}$

101. $\iint_D (x^2-y^2)\,dA$



$\int_{-1}^1\int_{y^4-1}^{1-y^4}(x^2-y^2)\,dx\,dy$

$= \int_{-1}^1 (\frac{1}{3}x^3 - xy^2\Big|_{y^4-1}^{1-y^4} dy$

$= \int_{-1}^1 (-\frac{2}{3}y^{12} + 2y^8 + 2y^6 - 2y^4 - 2y^2 + \frac{2}{3})\,dy$

$= (-\frac{2}{39}y^{13} + \frac{2}{9}y^9 + \frac{2}{7}y^7 - \frac{2}{5}y^5 - \frac{2}{3}y^3 + \frac{2}{3}y\Big|_{-1}^1$

$= (-\frac{2}{39} + \frac{2}{9} + \frac{2}{7} - \frac{2}{5} - \frac{2}{3} + \frac{2}{3}) - (\frac{2}{39} - \frac{2}{9} - \frac{2}{7} + \frac{2}{5} + \frac{2}{3} - \frac{2}{3})$

$= -\frac{4}{39} + \frac{4}{9} + \frac{4}{7} - \frac{4}{5}$

$= \frac{464}{4095}$

103. $z = 3x+y$ in Q1, above $y = x^7$ & $y = x$

$\int_0^1\int_{x^7}^{x}(3x+y)\,dy\,dx$

$= \int_0^1 (3xy + \frac{1}{2}y^2\Big|_{x^7}^{x} dx$

$= \int_0^1 (3x^2 + \frac{1}{2}x^2 - 3x^8 - \frac{1}{2}x^{14})\,dx$

$= (x^3 + \frac{1}{6}x^3 - \frac{1}{3}x^9 - \frac{1}{30}x^{15}\Big|_0^1$

$= (1 + \frac{1}{6} - \frac{1}{3} - \frac{1}{30}) - 0$

$= \frac{24}{30}$

$= \frac{4}{5}$

138. $f(x,y) = \sqrt[3]{x^2+y^2}$

$D = \{(r,\theta) \mid 0 \le r \le 1,\ \frac{\pi}{2} \le \theta \le \pi\}$

$\int_{\pi/2}^{\pi}\int_0^1 r^{2/3}\, r\,dr\,d\theta$

$= \int_{\pi/2}^{\pi}\int_0^1 r^{5/3}\,dr\,d\theta$

$= \int_{\pi/2}^{\pi}(\frac{3}{8}r^{8/3}\Big|_0^1 d\theta$

$= \int_{\pi/2}^{\pi}\frac{3}{8}\,d\theta$

$= (\frac{3}{8}\theta\Big|_{\pi/2}^{\pi}$

$= \frac{3\pi}{8} - \frac{3}{8}(\frac{\pi}{2})$

$= 3\pi/16$

139. $f(x,y) = x^4 + 2x^2y^2 + y^4$

$D = \{(r,\theta) \mid 3 \le r \le 4,\ \frac{\pi}{3} \le \theta \le \frac{2\pi}{3}\}$

$\int_{\pi/3}^{2\pi/3}\int_3^4 r^4\, r\,dr\,d\theta$

$= \int_{\pi/3}^{2\pi/3}\int_3^4 r^5\,dr\,d\theta$

$= \int_{\pi/3}^{2\pi/3}(\frac{1}{6}r^6\Big|_3^4 d\theta$

$= \int_{\pi/3}^{2\pi/3}(\frac{4096}{6} - \frac{729}{6})\,d\theta$

$= \int_{\pi/3}^{2\pi/3}\frac{3367}{6}\,d\theta$

$= \frac{3367}{6}\theta\Big|_{\pi/3}^{2\pi/3}$

$= \frac{3367}{6}(\frac{2\pi}{3} - \pi/3)$

$= \frac{3367\pi}{18}$

149. $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (x^2+y^2)^2 \, dx\, dy$

$\int_0^{\pi} \int_0^2 r^4 \, r\, dr\, d\theta$

$= \int_0^{\pi} \int_0^2 r^5 \, dr\, d\theta$

$= \int_0^{\pi} \left(\frac{1}{6} r^6 \Big|_0^2 \right) d\theta$

$= \int_0^{\pi} \frac{32}{3} \, d\theta$

$= \frac{32}{3} \theta \Big|_0^{\pi}$

$= \frac{32\pi}{3}$

153. D between polar axis & $r = 1+\cos\theta$ upper half

$\int_0^{\pi} \int_0^{1+\cos\theta} r \, dr\, d\theta$

$= \int_0^{\pi} \left(\frac{1}{2} r^2 \Big|_0^{1+\cos\theta} \right) d\theta$

$= \int_0^{\pi} \frac{1}{2}(1+\cos\theta)^2 \, d\theta$

$= \int_0^{\pi} \left(\frac{1}{2} + \cos\theta + \frac{1}{2}\cos^2\theta \right) d\theta$

$= \left(\frac{1}{2}\theta + \sin\theta + \frac{1}{4}\theta + \frac{\sin 2\theta}{8} \right) \Big|_0^{\pi}$

$= \left(\frac{1}{2}\pi + 0 + \frac{1}{4}\pi + 0 \right) - 0$

$= \frac{3\pi}{4}$

165. Cylindrical hole $d = 6$cm bored through sphere $r = 5$cm find Volume of resulting ring.

$V_{sphere} - V_{cylinder} = 2\int_0^{2\pi} \int_0^5 r\sqrt{25-r^2} \, dr\, d\theta - 8\int_0^{\pi/2} \int_0^3 r\sqrt{25-r^2} \, dr\, d\theta$

use u-sub for the integrands in terms of r

$= 2\int_0^{2\pi} \left(-\frac{1}{3}(25-r^2)^{3/2} \Big|_0^5 \right) d\theta - 8\int_0^{\pi/2} \left(-\frac{1}{3}(25-r^2)^{3/2} \Big|_0^3 \right) d\theta$

$= 2\int_0^{2\pi} 0 - \left(-\frac{1}{3}(25)^{3/2} \right) d\theta - 8\int_0^{\pi/2} \left(-\frac{1}{3}(16)^{3/2} + \frac{1}{3}(25)^{3/2} \right) d\theta$

$= 2\int_0^{2\pi} \frac{125}{3} \, d\theta - 8\int_0^{\pi/2} \left(\frac{125}{3} - \frac{64}{3} \right) d\theta$

$= 2\left(\frac{125}{3} \theta \Big|_0^{2\pi} \right) - 8\left(\frac{61}{3} \theta \Big|_0^{\pi/2} \right)$

$= \left[2\left(\frac{250\pi}{3} \right) - 0 \right] - \left[8\left(\frac{61\pi}{6} \right) - 0 \right]$

$= \frac{500\pi}{3} - \frac{244\pi}{3}$

$= \frac{256\pi}{3} \text{ cm}^3$