WP13 Solutions

381. $x = ue^{v} \quad y = e^{-v}$

$x_u = e^{v} \quad y_u = 0$

$x_v = ue^{v} \quad y_v = -e^{-v}$

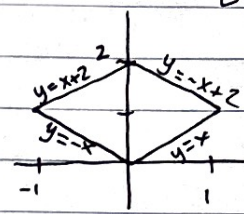
$J(u,v) = \begin{vmatrix} e^{v} & 0 \\ ue^{v} & -e^{-v} \end{vmatrix}$

$= e^{v}(e^{-v}) - 0(ue^{v})$

$J(u,v) = -1$

392. $y - x = u;\ x + y = v$

Evaluate $\iint_R e^{x+y}\, dA$



$\begin{cases} x = v - y \\ x = y - u \end{cases} \Rightarrow v - y = y - u \qquad x = v - \frac{v+u}{2}$

$2y = v + u \qquad x = \frac{2v - v - u}{2}$

$y = \frac{v+u}{2} \qquad x = \frac{v-u}{2}$

$J(u,v) = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

$\iint_R e^{x+y}\, dA = \int_0^2 \int_0^2 -\frac{1}{2} e^{v}\, du\, dv$

$= \int_0^2 -\frac{1}{2} e^{v} u \Big|_0^2 dv$

$= \int_0^2 -e^{v}\, dv$

$= -e^{v} \Big|_0^2$

$= 1 - e^{2}$

393. $y - x = u;\ x + y = v$

Evaluate $\iint_R \sin(x-y)\, dA$

from 392: $x = \frac{v-u}{2}$; $y = \frac{v+u}{2}$

and $J(u,v) = -\frac{1}{2}$

$\iint_R \sin(x-y)\, dA = \int_0^2 \int_0^2 -\frac{1}{2} \sin u\, du\, dv$

$= \int_0^2 \frac{1}{2} \cos u \Big|_0^2 dv$

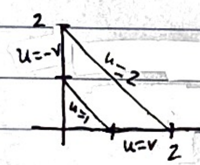
$= \int_0^2 \frac{1}{2} \cos 2 - \frac{1}{2}\, dv$

$= \frac{1}{2} v \cos 2 - \frac{1}{2} v \Big|_0^2$

$= \cos^2 - 1$

397. $u = x + y,\ v = x - y$

Evaluate $\iint_R (x^3 + 3x^2y + 3xy^2 + y^3)\, dA$



$x = u - y \qquad u - y = v + y \qquad x = u - \frac{u-v}{2}$

$x = v + y \qquad u - v = 2y \qquad x = \frac{2u - u + v}{2}$

$y = \frac{u-v}{2} \qquad x = \frac{u+v}{2}$

$J(u,v) = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

$\iint_R (x^3 + 3x^2y + 3xy^2 + y^3)\, dA = \iint_R (x+y)^3\, dA$

$= \int_1^2 \int_u^{-u} -\frac{1}{2} u^3\, dv\, du$

$= \int_1^2 -\frac{1}{2} v u^3 \Big|_u^{-u} du$

$= \int_1^2 \frac{1}{2} u^4 - (-\frac{1}{2} u^4)\, du$

$= \int_1^2 u^4\, du$

$= \frac{1}{5} u^5 \Big|_1^2$

$= \frac{32}{5} - \frac{1}{5}$

$= 31/5$