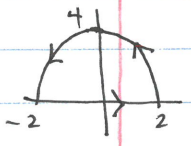
WP15 Solutions

147. $\int_C 2xy\,dx + (x+y)\,dy$

C: between $y=0$ & $y=4-x^2$ G



$$\int_{-2}^{2}\int_{0}^{4-x^2}(1+2x)\,dy\,dx$$

$$= \int_{-2}^{2} y(1+2x)\Big|_0^{4-x^2} dx$$

$$= \int_{-2}^{2}(4-x^2)(1+2x)\,dx$$

$$= \int_{-2}^{2}(-2x^3-x^2+8x+4)\,dx$$

$$= \left(-\tfrac{1}{2}x^4-\tfrac{1}{3}x^3+4x^2+4x\right)\Big|_{-2}^{2}$$

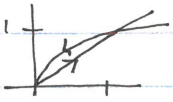
$$= \left(-8-\tfrac{8}{3}+16+8\right)-\left(-8+\tfrac{8}{3}+16-8\right)$$

$$= \tfrac{40}{3}-\tfrac{8}{3}$$

$$= 32/3$$

149. $\int_C \sin x\cos y\,dx + (xy\cos x\sin y)\,dy$

C: between $y=x$, $y=\sqrt{x}$ G



$$\int_0^1\int_x^{\sqrt{x}}(y-\sin x\sin y+\sin x\sin y)\,dy\,dx$$

$$= \int_0^1\int_x^{\sqrt{x}} y\,dy\,dx$$

$$= \int_0^1 \tfrac{1}{2}y^2\Big|_x^{\sqrt{x}}\,dx$$

$$= \int_0^1\left(\tfrac{1}{2}x-\tfrac{1}{2}x^2\right)dx$$

$$= \left(\tfrac{1}{4}x^2-\tfrac{1}{6}x^3\right)\Big|_0^1$$

$$= \left(\tfrac{1}{4}-\tfrac{1}{6}\right)-0$$

$$= \tfrac{1}{12}$$

150. $\int_C xy\,dx + (x+y)\,dy$

C: between $x^2+y^2=1$ & $x^2+y^2=9$ G

$$\iint_C (1-x)\,dA = \int_0^{2\pi}\int_1^3 (1-r\cos\theta)\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\tfrac{1}{2}r^2-\tfrac{1}{3}r^3\cos\theta\Big|_1^3\,d\theta$$

$$= \int_0^{2\pi}\left[\left(\tfrac{9}{2}-9\cos\theta\right)-\left(\tfrac{1}{2}-\tfrac{1}{3}\cos\theta\right)\right]d\theta$$

$$= \left(\tfrac{9}{2}\theta+9\sin\theta-\tfrac{1}{2}\theta-\tfrac{1}{3}\sin\theta\right)\Big|_0^{2\pi}$$

$$= (9\pi+0)-\pi-0-0$$

$$= 8\pi$$

174. $F(x,y) = xy\,\hat{i}+(x+y)\,\hat{j}$; $C: x^2+y^2=4$

$$W = \iint_C (1-x)\,dA = \int_0^{2\pi}\int_0^2 (1-r\cos\theta)\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\tfrac{1}{2}r^2-\tfrac{1}{3}r^3\cos\theta\Big|_0^2\,dr\,d\theta$$

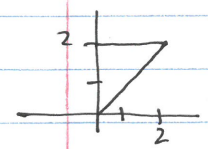
$$= \int_0^{2\pi}\left(2-\tfrac{8}{3}\cos\theta\right)d\theta$$

$$= 2\theta+\tfrac{8}{3}\sin\theta\Big|_0^{2\pi}$$

$$= (4\pi+0)-0$$

$$= 4\pi$$

194. $\int_C \sin(x^3)\,dx + 2ye^{x^2}\,dy$ C: triangle (0,0) (2,2) (0,2)



$$= \int_0^2\int_x^2 (4xye^{x^2}-0)\,dy\,dx$$

$$= \int_0^2\int_0^y 4xye^{x^2}\,dx\,dy$$

$$= \int_0^2 2ye^{x^2}\Big|_0^y\,dy$$

$$= \int_0^2 (2ye^{y^2}-2y)\,dy$$

$$= (e^{y^2}-y^2)\Big|_0^2$$

$$= (e^4-4)-(1-0)$$

$$= e^4-5$$