WP 2 Solutions

247. $P(1,-2,3)$ $V=\langle 1,2,3\rangle$

A) $x=1+t,\ y=-2+2t,\ z=3+3t$

B) $x-1=\frac{y+2}{2}=\frac{z-3}{3}$

C) If this intersects the xy-plane then $z=0$ which means $t=-1$
so $(0,-4,0)$

251. $x=1+t,\ y=3+t,\ z=5+4t$

A) $P(1,3,5)$ and $v=\langle 1,1,4\rangle$

B) $\overrightarrow{OP}=\langle 0-1,0-3,0-5\rangle$
$=\langle -1,-3,-5\rangle$

$$\overrightarrow{OP}\times\vec{V}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&-3&-5\\1&1&4\end{vmatrix}$$

$=(-12+5)\hat{i}-(-4+5)\hat{j}+(-1+3)\hat{k}$

$=-7\hat{i}-\hat{j}+2\hat{k}$

$$d=\frac{\sqrt{(-7)^2+(-1)^2+2^2}}{\sqrt{1^2+1^2+4^2}}$$

$d=\frac{\sqrt{54}}{\sqrt{18}}$

$d=\sqrt{3}$

257. $P(3,1,0)$ $Q(1,4,-3)$ want $\perp$ to $x=3t,\ y=3+8t,\ z=-7+6t$

$V=\langle 1-3,4-1,-3-0\rangle$
$=\langle -2,3,-3\rangle$

let v_1 be the vector from the line
$v_1=\langle 3,8,6\rangle$

so $v\cdot v_1=-2(3)+3(8)+(-3)(6)$
$=-6+24-18$
$=0$

So these are perpendicular

269. $P(1,2,3)$, $n=\langle 1,2,3\rangle$

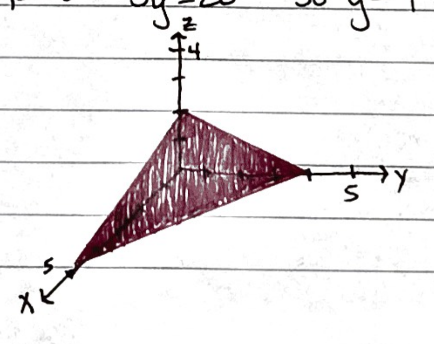
A) $1(x-1)+2(y-2)+3(z-3)=0$

B) $x-1+2y-4+3z-9=0$
$x+2y+3z-14=0$

271. $4x+5y+10z-20=0$

A) $n=4\hat{i}+5\hat{j}+10\hat{k}$

B) $x,y=0:\ 10z=20$ so $z=2$ $(0,0,2)$
$y,z=0:\ 4x=20$ so $x=5$ $(5,0,0)$
$x,z=0:\ 5y=20$ so $y=4$ $(0,4,0)$



279. Show $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ is parallel to $x-2y+z=6$

From the line: $x=2t+1, y=3t-1, z=4t+2$

$(2t+1)-2(3t-1)+(4t+2)=6$

$2t+1-6t+2+4t+2=6$

$5=6$ False

So there are no points of intersection, hence the line is parallel to the plane.

285. $P(-1,2,1)$ perpendicular intersection of $x+y-z-2=0$ and $2x-y+3z-1=0$.

$n_1=\langle 1,1,-1\rangle$ and $n_2=\langle 2,-1,3\rangle$

So the normal vector of the intersection is $n=n_1\times n_2$

$$n=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$=(3-1)\hat{i}-(3+2)\hat{j}+(-1-2)\hat{k}$$

$n=\langle 2,-5,-3\rangle$

The plane through P is

$2(x+1)-5(y-2)-3(z-1)=0$

$2x+2-5y+10-3z+3=0$

$2x-5y-3z+15=0$

313. Hyperboloid of two sheets

b. $\frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{12} = 1$

314. Ellipsoid

c. $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{12} = 1$

315. Elliptic Paraboloid

d. $z^2 = 4x^2 + 3y^2$

316. Hyperbolic Paraboloid

e. $z = 4x^2 - y^2$

317. Hyperboloid of one sheet

a. $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{12} = 1$

318. Elliptic Cone

f. $4x^2 + y^2 - z^2 = 0$

323. $5y = x^2 - z^2$

$y = \frac{x^2}{5} - \frac{z^2}{5}$

hyperbolic paraboloid with y-axis as the axis of the surface

327. $x^2 + 5y^2 - 8z^2 = 0$

$\frac{x^2}{40} + \frac{y^2}{8} - \frac{z^2}{5} = 0$

elliptic cone with the z-axis as the axis of the surface

339. $x^2 + 2z^2 + 6x - 8z + 1 = 0$

A) $x^2 + 6x + 2(z^2 - 4z) = -1$

$(x^2 + 6x + 9) + 2(z^2 - 4z + 4) = -1 + 9 + 8$

$(x+3)^2 + 2(z-2)^2 = 16$

$\frac{(x+3)^2}{16} + \frac{(z-2)^2}{8} = 1$

B) Cylinder centered at $(-3, 2)$ with rullings parallel to the y-axis.

345. $A(2,0,0)$, $B(0,0,1)$, $C(\frac{1}{2}, \sqrt{11}, \frac{1}{2})$

Ellipsoid cenentered at origin

$\frac{x^3}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

at A: $\frac{4}{a^2} = 1$ so $a^2 = 4$

at B: $\frac{1}{c^2} = 1$ so $c^2 = 1$

at C: $\frac{1/4}{4} + \frac{11}{b^2} + \frac{1/4}{1} = 1$

$\frac{1}{16} + \frac{11}{b^2} + \frac{1}{4} = 1$

$b^2 + 176 + 4b^2 = 16b^2$

$176 = 11b^2$

$16 = b^2$

so $\frac{x^2}{4} + \frac{y^2}{16} + z^2 = 1$

363. $(4, \frac{\pi}{6}, 3)$

$x = 4\cos \pi/6 = 2\sqrt{3}$
$y = 4\sin \pi/6 = 2$
$z = 3$
so $(2\sqrt{3}, 2, 3)$

369. $(3, -3, 7)$

$r^2 = 3^2 + (-3)^2$
$= 18$
$r = 3\sqrt{2}$

$\tan\theta = \frac{-3}{3}$
$\theta = \tan^{-1}(-1)$
$\theta = -\pi/4$

$z = 7$
so $(3\sqrt{2}, -\pi/4, 7)$

387. $(12, -\pi/4, \pi/4)$

$x = 12\sin \pi/4 \cos(-\pi/4)$
$= 12(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2})$
$= 6$
$y = 12\sin \pi/4 \sin(-\pi/4)$
$= 12(\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2})$
$= -6$
$z = 12\cos \pi/4$
$= 12(\frac{\sqrt{2}}{2})$
$= 6\sqrt{2}$
so $(6, -6, 6\sqrt{2})$

391. $(0, 3, 0)$

$\rho^2 = 0^2 + 3^2 + 0^2$
$= 9$
$\rho = 3$
$\tan\theta = \frac{3}{0}$
$\theta = \pi/2$ or $90°$
$\phi = \arccos\left(\frac{0}{\sqrt{0^2+3^2+0^2}}\right)$
$= \arccos(0)$
$= \pi/2$ or $90°$
so $(3, 90°, 90°)$ or $(3, \frac{\pi}{2}, \frac{\pi}{2})$

399. $x^2 + y^2 - 3z^2 = 0,\ z \neq 0$

$x^2 + y^2 = 3z^2$
$\rho^2\sin^2\phi = 3\rho^2\cos^2\phi$
$\tan^2\phi = 3$
$\tan\phi = \sqrt{3}$
$\phi = \pi/3$
Elliptical Cone

401. $z = 6$
$\rho\cos\phi = 6$
this is a plane at
$z = 6$.

405. $(3, \pi/2, 3)$

$\rho = \sqrt{3^2+3^2} = 3\sqrt{2}$
$\theta = \pi/2$
$\phi = \arccos\left(\frac{3}{\sqrt{3^2+3^2}}\right)$
$= \arccos(\sqrt{2}/2) = \pi/4$
so $(3\sqrt{2}, \pi/2, \pi/4)$

409. $(8, \pi/3, \pi/2)$

$r = 8 \sin \pi/2$
$\quad = 8$
$\theta = \pi/3$
$z = 8 \cos \pi/2$
$\quad = 0$
so $(8, \pi/3, 0)$

411. vertex at origin, enclosed by cube w/ edge length a, $a > 0$.

Rectangular System
$\{(x, y, z) \mid 0 \le x, y, z \le a\}$

413. Inside of sphere $x^2 + y^2 + z^2 = 9$ and outside cylinder $(x - 3/2)^2 + y^2 = 9/4$

Cylindrical System
$\{(r, \theta, z) \mid r^2 + z^2 \le 9,\ r \ge 0,\ \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\}$