WP3 Solutions

11. $\lim_{t\to e^2} \langle t\ln(t), \frac{\ln t}{t^2}, \sqrt{\ln(t^2)} \rangle$

$= \langle \lim_{t\to e^2} t\ln(t), \lim_{t\to e^2} \frac{\ln t}{t^2}, \lim_{t\to e^2} \sqrt{\ln(t^2)} \rangle$
$= \langle e^2\ln(e^2), \frac{\ln e^2}{e^4}, \sqrt{\ln(e^4)} \rangle$
$= \langle 2e^2, \frac{2}{e^4}, 2 \rangle$

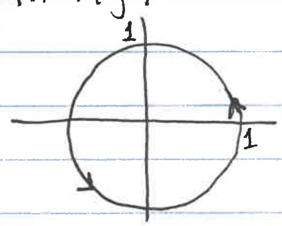
15. $r(t) = \langle t^2, \tan t, \ln t \rangle$

t^2 is defined for all real numbers
$\tan t$ is defined for $(-\frac{\pi}{2}+n\pi, \frac{\pi}{2}+n\pi)$
$\ln t$ is defined for $t \in (0,\infty)$
so $D_r: t>0$ where $t \neq \frac{(2n+1)\pi}{2}$ for $n \in \mathbb{Z}$

19. $r(t) = \langle \cos t, t, \sin t \rangle$

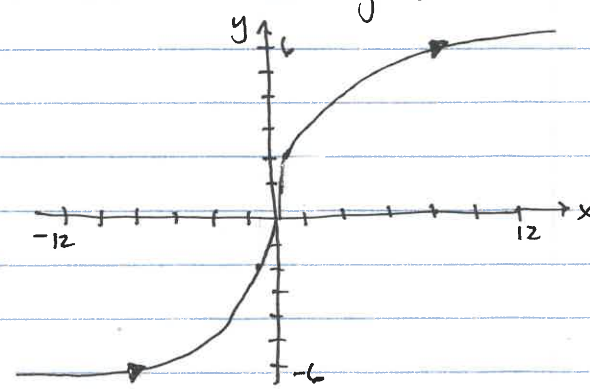
t	$r(t)$
0	$\hat{i}$
$\pi/6$	$\frac{\sqrt{3}}{2}\hat{i} + \frac{\pi}{6}\hat{j} + \frac{1}{2}\hat{k}$
$\pi/4$	$\frac{\sqrt{2}}{2}\hat{i} + \pi/4\,\hat{j} + \frac{\sqrt{2}}{2}\hat{k}$
$\pi/3$	$1/2\,\hat{i} + \pi/3\,\hat{j} + \frac{\sqrt{3}}{2}\hat{k}$
$\pi/2$	$\pi/2\,\hat{j} + \hat{k}$

in x,y plane



23. $r(t) = t^3\hat{i} + 2t\hat{j}$

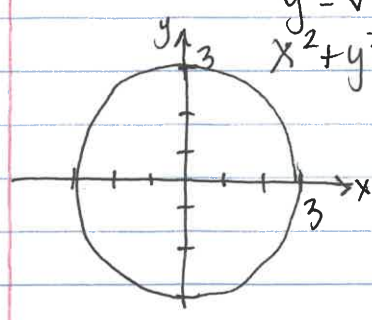
$x = t^3, y = 2t$
$t = \sqrt[3]{x}$ so $y = 2\sqrt[3]{x}$



25. $r(t) = 3\cos t\,\hat{i} + 3\sin t\,\hat{j}$

$x = 3\cos t$
$\frac{x}{3} = \cos t$
$t = \cos^{-1}(\frac{x}{3})$

$y = 3\sin t$
$y = 3\sin(\cos^{-1}\frac{x}{3})$
$y = 3\left(\frac{\sqrt{9-x^2}}{3}\right)$
$y = \sqrt{9-x^2}$
$x^2+y^2=9$



33. $r(t) = (50e^{-t}\cos t)\hat{i} + (50e^{-t}\sin t)\hat{j} + (5-5e^{-t})\hat{k}$

$r(0) = (50e^0\cos 0)\hat{i} + (50e^0\sin 0)\hat{j} + (5-5e^0)\hat{k}$
$= 50\hat{i} + 0\hat{j} + 0\hat{k}$
$= 50\hat{i}$

so the initial point is $(50,0,0)$

41. $r(t) = t^3\hat{i} + 3t^2\hat{j} + \frac{t^3}{6}\hat{k}$

$r'(t) = 3t^2\hat{i} + 6t\hat{j} + \frac{t^2}{2}\hat{k}$

43. $r(t) = e^{-t}\hat{i} + \sin(3t)\hat{j} + 10\sqrt{t}\,\hat{k}$

$r'(t) = -e^{-t}\hat{i} + 3\cos(3t)\hat{j} + \frac{5}{\sqrt{t}}\hat{k}$

47. $r(t) = \frac{1}{t+1}\hat{i} + \arctan(t)\hat{j} + \ln t^3\hat{k}$

$r'(t) = \frac{-1}{(t+1)^2}\hat{i} + \frac{1}{1+t^2}\hat{j} + \frac{3}{t}\hat{k}$

55. $r(t) = 6\hat{i} + \cos(3t)\hat{j} + 3\sin(4t)\hat{k}$; $0 \le t \le 2\pi$

$r'(t) = 0\hat{i} - 3\sin(3t)\hat{j} + 12\cos(4t)\hat{k}$

$\|r'(t)\| = \sqrt{9\sin^2(3t) + 144\cos^2(4t)}$

$T(t) = \dfrac{-3\sin(3t)\hat{j} + 12\cos(4t)\hat{k}}{\sqrt{9\sin^2(3t) + 144\cos^2(4t)}}$

63. $r(t) = -3t^5\hat{i} + 5t\hat{j} + 2t^2\hat{k}$

$r'(t) = -15t^4\hat{i} + 5\hat{j} + 4t\hat{k}$

$r''(t) = -60t^3\hat{i} + 4\hat{k}$

$r' \cdot r'' = (-15t^4)(-60t^3) + 5(0) + 4t(4)$

$= 900t^7 + 16t$

67. $r(t) = b\cos(wt)\hat{i} + b\sin(wt)\hat{j}$

$v(t) = -bw\sin(wt)\hat{i} + bw\cos(wt)\hat{j}$

$v \cdot r = -bw\sin(wt) \cdot b\cos(wt)$

$+ bw\cos(wt) \cdot b\sin(wt)$

$= -b^2w\sin(wt)\cos(wt)$

$+ b^2w\sin(wt)\cos(wt)$

$= 0$

so $v(t)$ is orthogonal to $r(t)$.

71. $\int_0^3 \|t\hat{i} + t^2\hat{j}\|\,dt$

$\int_0^3 \sqrt{t^2 + (t^2)^2}\,dt$

$= \int_0^3 t\sqrt{1+t^2}\,dt$

let $u = 1+t^2$ so $du = 2t\,dt$

when $t=0 \to u=1$ and $t=3 \to u=10$

$= \frac{1}{2}\int_1^{10} u^{1/2}\,du$

$= \frac{1}{3}u^{3/2}\Big|_1^{10}$

$= \frac{1}{3}(10)^{3/2} - \frac{1}{3}(1)^{3/2}$

$= \dfrac{10\sqrt{10} - 1}{3}$

75. $r(t) = \langle t + \cos t,\ t - \sin t \rangle$

$v(t) = \langle 1 - \sin t,\ 1 - \cos t \rangle$

$\|v(t)\| = \sqrt{(1-\sin t)^2 + (1-\cos t)^2}$

$= \sqrt{1 - 2\sin t + 1 - 2\cos t + 1}$

$= \sqrt{3 - 2(\sin t + \cos t)}$