

WP5 Solutions

119. $z = \sin(3x)\cos(3y)$; $\frac{\partial z}{\partial y}$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \sin(3x) \cdot 3(-\sin(3y)) \\ &= -3\sin(3x)\sin(3y)\end{aligned}$$

121. $z = \ln(x^6 + y^4)$; $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{1}{x^6 + y^4} (6x^5) \\ &= \frac{6x^5}{x^6 + y^4} \\ \frac{\partial z}{\partial y} &= \frac{1}{x^6 + y^4} (4y^3) \\ &= \frac{4y^3}{x^6 + y^4}\end{aligned}$$

125. $z = \tan(2x - y)$; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

$$\begin{aligned}\frac{\partial z}{\partial x} &= \sec^2(2x - y) \cdot 2 \\ &= 2\sec^2(2x - y) \\ \frac{\partial z}{\partial y} &= \sec^2(2x - y) (-1) \\ &= -\sec^2(2x - y)\end{aligned}$$

149. $z = e^x \sin y$

$$\begin{aligned}\frac{\partial z}{\partial x} &= e^x \sin y \\ \frac{\partial^2 z}{\partial x^2} &= e^x \sin y \\ \frac{\partial z}{\partial y} &= e^x \cos y \\ \frac{\partial^2 z}{\partial y^2} &= -e^x \sin y\end{aligned}$$

$$\begin{aligned}\text{So } \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} &= e^x \sin y - e^x \sin y \\ &= 0.\end{aligned}$$

157. $f(x, y, t) = e^{-2t} \sin x \sin y$

$$\begin{aligned}\frac{\partial f}{\partial t} &= -2e^{-2t} \sin x \sin y \\ \frac{\partial f}{\partial x} &= e^{-2t} \cos x \sin y \\ \frac{\partial^2 f}{\partial x^2} &= -e^{-2t} \sin x \sin y \\ \frac{\partial f}{\partial y} &= e^{-2t} \sin x \cos y \\ \frac{\partial^2 f}{\partial y^2} &= -e^{-2t} \sin x \sin y \\ \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} &= -e^{-2t} \sin x \sin y + (-e^{-2t} \sin x \sin y) \\ &= -2e^{-2t} \sin x \sin y \\ &= \frac{\partial f}{\partial t} \quad \checkmark\end{aligned}$$

161. $f(x, y) = 200x^{0.7}y^{0.3}$; $x = 500$, $y = 1000$

$$\begin{aligned}\frac{\partial f}{\partial x} &= 200(0.7)x^{-0.3}y^{0.3} \\ \frac{\partial f}{\partial x} \Big|_{(500, 1000)} &= \frac{140(1000)^{0.3}}{500^{0.3}}\end{aligned}$$

$$= 140(2)^{0.3}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= 200(0.3)x^{0.7}y^{-0.7} \\ \frac{\partial f}{\partial y} \Big|_{(500, 1000)} &= \frac{60(500)^{0.7}}{1000^{0.7}} \\ &= \frac{60}{2^{0.7}}\end{aligned}$$