

Wp6 Solutions

217. $w = 5x^2 + 2y^2$, $x = -3s + t$, $y = s - 4t$, $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$

$$\begin{aligned}\frac{\partial w}{\partial s} &= \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} \\ &= 10x(-3) + 4y(1) \\ &= -30x + 4y\end{aligned}$$

$$\begin{aligned}\frac{\partial w}{\partial t} &= \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} \\ &= 10x(1) + 4y(-4) \\ &= 10x - 16y\end{aligned}$$

223. $f(x, y) = xy$, $x = 1 - \sqrt{t}$, $y = 1 + \sqrt{t}$

$$f_x = y \quad f_y = x$$

$$x_t = -\frac{1}{2\sqrt{t}} \quad y_t = \frac{1}{2\sqrt{t}}$$

$$\begin{aligned}\frac{\partial f}{\partial t} &= f_x \cdot x_t + f_y \cdot y_t \\ &= (1 + \sqrt{t})\left(-\frac{1}{2\sqrt{t}}\right) + (1 - \sqrt{t})\left(\frac{1}{2\sqrt{t}}\right) \\ &= -\frac{1}{2\sqrt{t}} - \frac{1}{2} + \frac{1}{2\sqrt{t}} - \frac{1}{2} \\ &= -1\end{aligned}$$

243. $z = \frac{x}{y}$, $x = 2\cos u$, $y = 3\sin v$, $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

$$\begin{aligned}\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} \\ &= \frac{1}{y}(-2\sin u) \\ &= \frac{-2\sin u}{3\sin v}\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial v} &= \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v} \\ &= -\frac{x}{y^2}(3\cos v) \\ &= \frac{-2\cos u \cos v}{3\sin^2 v}\end{aligned}$$

245. $z = xye^{xy}$, $x = r\cos\theta$, $y = r\sin\theta$, $\frac{\partial z}{\partial r}$, $\frac{\partial z}{\partial \theta}$, $r = 2$, $\theta = \frac{\pi}{6}$

$$\begin{aligned}\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r} \\ &= (ye^{xy} + xy \cdot \frac{1}{y}e^{xy})\cos\theta \\ &\quad + (xe^{xy} + xy \cdot \left(-\frac{x}{y^2}\right)e^{xy})\sin\theta \\ \frac{\partial z}{\partial r}\bigg|_{(2, \frac{\pi}{6})} &= (e^{\sqrt{3}} + \sqrt{3}e^{\sqrt{3}})\frac{\sqrt{3}}{2} \\ &\quad + (\sqrt{3}e^{\sqrt{3}} + \sqrt{3}(-\sqrt{3})e^{\sqrt{3}})\frac{1}{2} \\ &= \sqrt{3}e^{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial \theta} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial \theta} \\ &= (ye^{xy} + xy \cdot \frac{1}{y}e^{xy})(-r\sin\theta) \\ &\quad + (xe^{xy} + xy \cdot \left(-\frac{x}{y^2}\right)e^{xy})(r\cos\theta) \\ \frac{\partial z}{\partial \theta}\bigg|_{(2, \frac{\pi}{6})} &= (e^{\sqrt{3}} + \sqrt{3}e^{\sqrt{3}})(-1) \\ &\quad + (\sqrt{3}e^{\sqrt{3}} + \sqrt{3}(-\sqrt{3})e^{\sqrt{3}})\sqrt{3} \\ &= (2 - 4\sqrt{3})e^{\sqrt{3}}\end{aligned}$$

$$251. V(x,y) = \pi x^2 y, x = \frac{1}{2}t, y = \frac{1}{3}t$$

$$\begin{aligned}\frac{\partial V}{\partial t} &= \frac{\partial V}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial V}{\partial y} \cdot \frac{\partial y}{\partial t} \\ &= (2\pi xy) \left(\frac{1}{2}\right) + (\pi x^2) \frac{1}{3} \\ &= \pi xy + \frac{\pi x^2}{3}\end{aligned}$$

$$\begin{aligned}\left. \frac{\partial V}{\partial t} \right|_{(2,5)} &= \pi(2)5 + \frac{\pi(4)}{3} \\ &= 10\pi + \frac{4\pi}{3} \\ &= \frac{34\pi}{3}\end{aligned}$$

$$257. x = \sqrt{1+t}, y = 2 + \frac{1}{3}t$$

$$T_x(2,3) = 4 \quad T_y(2,3) = 3$$

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial t} \\ &= T_x \left(\frac{1}{2\sqrt{1+t}} \right) + T_y \cdot \frac{1}{3}\end{aligned}$$

$$\begin{aligned}\left. \frac{\partial T}{\partial t} \right|_{t=3} &= 4 \left(\frac{1}{2\sqrt{4}} \right) + 3 \left(\frac{1}{3} \right) \\ &= 1 + 1 \\ &= 2^\circ/\text{sec}\end{aligned}$$