WP6 Solutions

217. $w=5x^2+2y^2$, $x=-3s+t$, $y=s-4t$, $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$

$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s}=10x(-3)+4y(1)=-30x+4y$$

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t}=10x(1)+4y(-4)=10x-16y$$

223. $f(x,y)=xy$, $x=1-\sqrt{t}$, $y=1+\sqrt{t}$

$f_x=y \qquad f_y=x$

$x_t=-\frac{1}{2\sqrt{t}} \qquad y_t=\frac{1}{2\sqrt{t}}$

$$\frac{\partial f}{\partial t}=f_x\cdot x_t+f_y\cdot y_t=(1+\sqrt{t})\left(-\frac{1}{2\sqrt{t}}\right)+(1-\sqrt{t})\left(\frac{1}{2\sqrt{t}}\right)=-\frac{1}{2\sqrt{t}}-\frac{1}{2}+\frac{1}{2\sqrt{t}}-\frac{1}{2}=-1$$

243. $z=\frac{x}{y}$, $x=2\cos u$, $y=3\sin v$, $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u}=\frac{1}{y}(-2\sin u)=\frac{-2\sin u}{3\sin v}$$

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}=-\frac{x}{y^2}(3\cos v)=-\frac{2\cos u\cos v}{3\sin^2 v}$$

245. $z=xye^{x/y}$, $x=r\cos\theta$, $y=r\sin\theta$ $\frac{\partial z}{\partial r}$, $\frac{\partial z}{\partial \theta}$, $r=2$, $\theta=\pi/6$

$$\frac{\partial z}{\partial r}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}=\left(ye^{x/y}+xy\cdot\frac{1}{y}e^{x/y}\right)\cos\theta+\left(xe^{x/y}+xy\left(\frac{-x}{y^2}\right)e^{x/y}\right)\sin\theta$$

$$\left.\frac{\partial z}{\partial r}\right|_{(2,\frac{\pi}{6})}=\left(e^{\sqrt{3}}+\sqrt{3}e^{\sqrt{3}}\right)\frac{\sqrt{3}}{2}+\left(\sqrt{3}e^{\sqrt{3}}+\sqrt{3}(-\sqrt{3})e^{\sqrt{3}}\right)\frac{1}{2}=\sqrt{3}e^{\sqrt{3}}$$

$$\frac{\partial z}{\partial \theta}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \theta}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \theta}=\left(ye^{x/y}+xy\frac{1}{y}e^{x/y}\right)(-r\sin\theta)+\left(xe^{x/y}+xy\left(-\frac{x}{y^2}\right)e^{x/y}\right)(r\cos\theta)$$

$$\left.\frac{\partial z}{\partial \theta}\right|_{(2,\frac{\pi}{6})}=\left(e^{\sqrt{3}}+\sqrt{3}e^{\sqrt{3}}\right)(-1)+\left(\sqrt{3}e^{\sqrt{3}}+\sqrt{3}(-\sqrt{3}e^{\sqrt{3}})\right)\sqrt{3}=(2-4\sqrt{3})e^{\sqrt{3}}$$

251. $V(x,y) = \pi x^2 y$, $x = \frac{1}{2}t$, $y = \frac{1}{3}t$

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial V}{\partial y}\cdot\frac{\partial y}{\partial t}$$
$$= (2\pi xy)\left(\tfrac{1}{2}\right) + (\pi x^2)\tfrac{1}{3}$$
$$= \pi xy + \frac{\pi x^2}{3}$$
$$\left.\frac{\partial V}{\partial t}\right|_{(2,5)} = \pi(2)5 + \frac{\pi(4)}{3}$$
$$= 10\pi + \frac{4\pi}{3}$$
$$= \frac{34\pi}{3}$$

257. $x = \sqrt{1+t}$, $y = 2 + \frac{1}{3}t$

$T_x(2,3) = 4$ $\quad T_y(2,3) = 3$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial T}{\partial y}\cdot\frac{\partial y}{\partial t}$$
$$= T_x\left(\frac{1}{2\sqrt{1+t}}\right) + T_y\cdot\frac{1}{3}$$
$$\left.\frac{\partial T}{\partial t}\right|_{t=3} = 4\left(\frac{1}{2\sqrt{4}}\right) + 3\left(\frac{1}{3}\right)$$
$$= 1 + 1$$
$$= 2^\circ/\text{sec}$$