WP7 Solutions

171. $z = -9x^2 - 3y^2$; $P(2,1,-39)$

$f_x = -18x \qquad f_y = -6y$

$f_x(2,1) = -36 \qquad f_y(2,1) = -6$

tangent:

$z = -39 - 36(x-2) - 6(y-1)$

$z = -39 - 36x + 72 - 6y + 6$

$36x + 6y + z = 39$

181. $z = x^2 - 2xy + y^2$; $P(1,2,1)$

$f_x = 2x - 2y \qquad f_y = -2x + 2y$

$f_x(1,2) = -2 \qquad f_y(1,2) = 2$

tangent:

$z = 1 - 2(x-1) + 2(y-2)$

$z = 1 - 2x + 2 + 2y - 4$

$2x + 2y + z = -1$

196. $V(r,h) = \pi r^2 h$

$V_r = 2\pi rh$

$V_h = \pi r^2$

$dV = 2\pi rh\,dr + \pi r^2\,dh$

197. $dV = 2\pi rh\,dr + \pi r^2\,dh$

$d = 8\text{cm}, \; h = 12\text{cm}, \; w = 0.04\text{cm}$

$dV = 2\pi(4)(12)(.04) + \pi(4^2)(.08)$

$= 3.84\pi + 1.28\pi$

$= 5.12\pi\,\text{cm}^3$ OR $16.08\,\text{cm}^3$

203. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$; $R_1 = 7\Omega$; $R_2 = 6\Omega$

and $\frac{\partial}{\partial r} = 0.05\Omega$

$R = \frac{R_1 R_2}{R_1 + R_2}$

$\frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1+R_2)^2} \qquad \frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1+R_2)^2}$

$dR\big|_{(7,6)} = \frac{6^2}{(7+6)^2}(0.05) + \frac{7^2}{(7+6)^2}(0.05)$

$= \frac{(36+49)(0.05)}{13^2}$

$= \frac{85(.05)}{169}$

$dR = 0.025$

207. $f(x,y) = x\sqrt{y} \qquad P(1,4)$

$f_x = \sqrt{y} \qquad f_y = \frac{x}{2\sqrt{y}} \qquad f(1,4) = 1\sqrt{4} = 2$

$f_x(1,4) = 2 \qquad f_y(1,4) = 1/4$

$L(1,4) = 2 + 2(x-1) + \frac{1}{4}(y-4)$

$= 2 + 2x - 2 + \frac{1}{4}y - 1$

$= 2x + \frac{1}{4}y - 1$

269. $f(x,y) = e^x \cos y$ $u = \langle 0,1 \rangle$ $P(0, \pi/2)$

$f_x = e^x \cos y$ $f_y = -e^x \sin y$

$f_x(0, \pi/2) = 0$ $f_y(0, \pi/2) = -1$

$D_u f(0, \pi/2) = 0(0) - 1(1)$

$= -1$

281. $f(x,y,z) = xy + yz + xz$; $P(1,2,3)$

$f_x = y + z$ $f_y = x + z$ $f_z = y + x$

$f_x(1,2,3) = 5$ $f_y(1,2,3) = 4$ $f_z(1,2,3) = 3$

$\nabla f = \langle 5,4,3 \rangle$

291. $f(x,y) = xe^y - \ln(x)$; $P(-3,0)$

$f_x = e^y - \frac{1}{x}$ $f_y = xe^y$

$f_x(-3,0) = 4/3$ $f_y(-3,0) = -3$

$\nabla f = \langle \frac{4}{3}, -3 \rangle$

307. $V(x,y,z) = 5x^2 - 3xy + xyz$

a) $P(3,4,5)$; $u = \langle 1,1,-1 \rangle$

$V_x = 10x - 3y + yz$ $V_x(3,4,5) = 38$

$V_y = -3x + xz$ $V_y(3,4,5) = 6$

$V_z = xy$ $V_z(3,4,5) = 12$

$\frac{u}{\|u\|} = \frac{\langle 1,1,-1 \rangle}{\sqrt{1^2+1^2+(-1)^2}} = \frac{\sqrt{3}}{3}\langle 1,1,-1 \rangle$

$D_u V(3,4,5) = 38\left(\frac{\sqrt{3}}{3}\right) + 6\left(\frac{\sqrt{3}}{3}\right) + 12(-1)$

$= \frac{32\sqrt{3}}{3}$

b) Which direction does the voltage change most rapidly?

from part a $V_x(3,4,5) = 38$

$V_y(3,4,5) = 6$

$V_z(3,4,5) = 12$

so $\nabla V = \langle 38, 6, 12 \rangle$

c) max rate at that point

$\|\nabla V\| = \sqrt{38^2 + 6^2 + 12^2}$

$= \sqrt{1624}$

$= 2\sqrt{406}$