WP8 Solutions

313. $f(x,y) = 15x^3 - 3xy + 15y^3$

$f_x = 45x^2 - 3y$
$f_y = -3x + 45y^2$
$f_x = 0 \Rightarrow 45x^2 = 3y$
$15x^2 = y$
$f_y = 0 \Rightarrow 45y^2 - 3x = 0$

$45(15x^2)^2 - 3x = 0$
$3x(3375x^3 - 1) = 0$
$3x = 0 \quad x^3 = \frac{1}{3375}$
$x = 0 \quad x = \frac{1}{15}$

$x=0: 15(0)^2 = y \Rightarrow y = 0$
$x = \frac{1}{15}: 15(\frac{1}{15})^2 = y \Rightarrow y = \frac{1}{15}$
cp: $(0,0)$ and $(\frac{1}{15}, \frac{1}{15})$

319. $f(x,y) = x^2y^2$

$f_x = 2xy^2$
$f_y = 2x^2y$
$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{matrix} 2xy^2 = 0 \\ 2x^2y = 0 \end{matrix}$
so $x = 0$ or $y = 0$

$f_{xx} = 2y^2$
$f_{yy} = 2x^2$
$f_{xy} = 4xy$

$D(0,0) = 0$; inconclusive
Since $x^2y^2 > 0$ for x, y not both zero and $x^2y^2 = 0$ for critical point
So $(0,0)$ is an absolute min

331. $f(x,y) = x^2 + 2y^2 - x^2y$

$f_x = 2x - 2xy$
$f_y = 4y - x^2$

$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{matrix} 2x - 2xy = 0 \\ 2x(1-y) = 0 \end{matrix}$
$x = 0$ or $1 = y$

$x=0: 4y - 0^2 = 0 \quad y=1: 4(1) - x^2 = 0$
$4y = 0 \quad 4 = x^2$
$y = 0 \quad \pm 2 = x$

cp: $(0,0)\ (2,1)\ (-2,1)$
$f_{xx} = 2 - 2y \quad f_{yy} = 4 \quad f_{xy} = -2x$
$D(0,0) = 8 \quad D(2,1) = D(-2,1) = -16$
$(0,0)$ is a local min
$(2,1)$ and $(-2,1)$ are saddle points

345. $f(x,y) = x^2 + y^2 - 2y + 1$; $R = \{(x,y) \mid x^2 + y^2 \le 4\}$

$f_x = 2x \quad f_y = 2y - 2$
$f_x = 0, f_y = 0 \Rightarrow x = 0, y = 1$
cp: $(0,1)$
$f_{xx} = 2, \ f_{yy} = 2, \ f_{xy} = 0$
$D(0,1) = 2(2) - 0^2 = 4$

On R: $x = 2\cos t, \ y = 2\sin t$
$h(t) = 4\cos^2 t + 4\sin^2 t - 2(2\sin t) + 1$
$= 5 - 4\sin t$
$h'(t) = -4\cos t \quad h' = 0$ when $t = \frac{\pi}{2}, \frac{3\pi}{2}$
$t = \frac{\pi}{2}: x = 0, y = 2 \Rightarrow f = 1$
$t = \frac{3\pi}{2}: x = 0, y = -2 \Rightarrow f = 9$
so $(0,1,0)$ is abs min and $(0,-2,9)$ is abs max

352. Open box w/o lid has volume 4 cuft
want to minimize Surface Area.

$S(l,w) = lw + 2lh + 2wh$

$V: lwh = 4$ so $h = \frac{4}{lw}$

$S = lw + \frac{8}{w} + \frac{8}{l}$

$S_l = w - 8/l^2$

$S_w = l - 8/w^2$

$\left.\begin{array}{l} S_l = 0 \\ S_w = 0 \end{array}\right\} \Rightarrow \begin{array}{l} w = 8/l^2 \\ l = 8/w^2 \end{array}$

$l = 8/(8/l)^2$

$0 = l^4 - 8l$

$0 = l(l^3 - 8)$

$l \neq 0$; $l = 2$

$l = 2$: $2 = \frac{8}{w^2}$

$w = 2$ and $h = 1$

$S_{ll} = \frac{16}{l^3}$ $S_{ww} = \frac{16}{w^3}$ $S_{lw} = 1$

$D(2,2) = 2(2) - 1^2$

$= 3$

So
2ft x 2ft x 1ft will give the minimum cardboard.

354. $2x - y + 2z = 16$ closest to origin

$z = 8 - x + \frac{1}{2}y$

dist: $d = \sqrt{(x-0)^2 + (y-0)^2 + (8 - x + \frac{1}{2}y - 0)^2}$

$f(x,y) = x^2 + y^2 + (8 - x + \frac{1}{2}y)^2$

$f_x = 2x + 2(8 - x + \frac{1}{2}y)(-1)$

$= 4x - y - 16$

$f_y = 2y + 2(8 - x + \frac{1}{2}y)(\frac{1}{2})$

$= -x + \frac{5}{2}y + 8$

$\left.\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\} \Rightarrow \begin{array}{l} 4x - y - 16 = 0 \\ -x + \frac{5}{2}y + 8 = 0 \end{array}$

$y = 4x - 16$

$-x + \frac{5}{2}(4x - 16) + 8 = 0$

$-x + 10x - 40 + 8 = 0$

$9x = 32$

$x = \frac{32}{9}$, $y = \frac{-16}{9}$

$z = 8 - \frac{32}{9} - 8/9 = 32/9$

$f_{xx} = 4$, $f_{yy} = 5/2$, $f_{xy} = -1$

$D(\frac{32}{9}, \frac{-16}{9}) = 4(\frac{5}{2}) - (-1)^2$

$= 9$ so min

The point $(\frac{32}{9}, -\frac{16}{9}, \frac{32}{9})$ is the closest point.