WP9 Solutions

359. $f(x,y,z)=xyz$; $x^2+2y^2+3z^2=6$
$g(x,y,z)=x^2+2y^2+3z^2-6$

$\nabla f = yz\hat{i}+xz\hat{j}+xy\hat{k}$
$\nabla g = 2x\hat{i}+4y\hat{j}+6z\hat{k}$
$yz\hat{i}+xz\hat{j}+xy\hat{k}=\lambda(2x\hat{i}+4y\hat{j}+6z\hat{k})$

$$\begin{cases} yz=2\lambda x \\ xz=4\lambda y \\ xy=6\lambda z \\ x^2+2y^2+3z^2=6 \end{cases}$$

$yz=2\lambda x \Rightarrow 2\lambda=\frac{yz}{x}$
$xz=2y\left(\frac{yz}{x}\right)$ and $xy=3z\left(\frac{yz}{x}\right)$
$x^2z=2y^2z$ $\quad x^2y=3yz^2$
$x^2=2y^2$ $\quad x^2=3z^2$

plugging into constraint

$x^2+x^2+x^2=6$ $\quad 2y^2=2$ $\quad 3z^2=2$
$3x^2=6$ $\quad y^2=1$ $\quad z^2=2/3$
$x^2=2$ $\quad y=\pm1$ $\quad z=\pm\sqrt{2/3}$
$x=\pm2$

So f is max at $\frac{2\sqrt{3}}{3}$ when an even # of x,y,z are neg.
f is min at $-\frac{2\sqrt{3}}{3}$ when an odd # of x,y,z are neg.

369. Minimize $f(x,y)=x^2+y^2$ on $xy=1$

$\nabla f = 2x\hat{i}+2y\hat{j}$
$\nabla g = y\hat{i}+x\hat{j}$
$2x\hat{i}+2y\hat{j}=\lambda(y\hat{i}+x\hat{j})$

$$\begin{cases} 2x=\lambda y \\ 2y=\lambda x \\ xy-1=0 \end{cases}$$

$2x=\lambda y \Rightarrow \lambda=\frac{2x}{y}$
$2y=\frac{2x}{y}(x)$
$2y^2=2x^2$
$y=\pm x$

$y=x$: $x(x)-1=0$
$x^2=1$
$x=\pm1$

$y=-x$: $x(-x)-1=0$
$-x^2=1$
no solution

$f(1,1)=1^2+1^2=2$ $\quad f(-1,-1)=(-1)^2+(-1)^2=2$

so f is a min at 2.

381. let x = length of box, y = width, z = height; $f(x,y,z)=xyz$ & $xy+2xz+2yz=12$

$\nabla f = yz\hat{i}+xz\hat{j}+xy\hat{k}$
$\nabla g = (y+2z)\hat{i}+(x+2z)\hat{j}+(2x+2y)\hat{k}$

$$\begin{cases} yz=\lambda(y+2z) \\ xz=\lambda(x+2z) \\ xy=\lambda(2x+2y) \\ xy+2xz+2yz=12 \end{cases} \rightarrow \begin{aligned} \lambda&=\frac{yz}{y+2z} \\ \lambda&=\frac{xz}{x+2z} \\ \lambda&=\frac{xy}{2x+2y} \end{aligned}$$

$\frac{yz}{y+2z}=\frac{xz}{x+2z}$
$xyz+2xz^2=xyz+2yz^2$
$2xz^2=2yz^2$
$2x=2y$
$x=y$

$\frac{yz}{y+2z}=\frac{xy}{2x+2y}$
$2xyz+2y^2z=xy^2+2xyz$
$2y^2z=xy^2$
$2z=x$

$xy+2xz+2yz=12$
$x^2+x^2+x^2=12$
$x^2=4 \rightarrow x=\pm2$

When x=2 the max volume is at 2ft x 2ft x 1ft.